$\mathrm{Gal}(MT(\mathbb{Z}), dR)(\mathbb{C}) \ni^{\exists} \mathrm{comp}_{dR,B} : \mathbb{C} \otimes_{\mathbb{Q}} M_B \cong \mathbb{C} \otimes_{\mathbb{Q}} M_{dR}$

$\downarrow$

$\mathbb{G}_m(\mathbb{C}) = \mathbb{C}^\times \ni 2\pi i$

well def'd up to $\mathrm{Gal}(MT(\mathbb{Z}), dR)(\mathbb{Q})$

$\forall M \in \mathrm{Obj}(MT(\mathbb{Z}))$

$\left(\mathbb{C}^\times \quad \int_{(0)} \frac{dz}{z} = 2\pi i \right)$

$\mathrm{Gal}(MT(\mathbb{Z}), dR) \xrightarrow{?} \mathrm{Aut}\, \pi_1^{dR}(V_{0,\vec{1}})$

K-theory

$K_{2n-1}(\mathbb{Z})_{\mathbb{Q}}$

$\mathrm{comp}_{dR,B} \longmapsto \Phi_{KZ}$

Drinfel'd assoc.

MZVs

enough small

$\mathrm{comp}_{dR,B}$ should satisfy defining eq's (← no one knows) of mo-var. $\mathrm{Gal}(MT(\mathbb{Z}), dR)$

meaning

generat… x^n =

$d_{n+3} = d_{n+1} + d$

comes from π_1^M

$\rightsquigarrow \mathrm{Gal}(MT(\mathbb{Z}), dR) \xrightarrow{?} \mathrm{Aut}\, \pi_1^{dR}(V_{0,…}$

B : $\mathbb{C}\otimes_{\mathbb{Q}} M_B \cong \mathbb{C}\otimes_{\mathbb{Q}} M_{dR}$

d $\quad {}^{t}M \in Obj(MT(\mathbb{Z}))$

$Gal(MT(\mathbb{Z}), dR)(\mathbb{Q})$

$\left(\mathbb{C}^\times \quad \int_{(0,1)} \frac{dt}{t} = 2\pi i \right)$

$Gal(MT(\mathbb{Z}), dR) \xrightarrow{?} Aut\, \pi_1^{dR}(V/\mathbb{Q}; \vec{01})$ — too big

K-theory $K_{2n-1}(\mathbb{Z})/\mathbb{Q}$

$comp_{dR,B} \longmapsto \Phi_{KZ}$ Drinfel'd assoc.

enough small

$comp_{dR,B}$ should satisfy defining eq's (← no one knows) of mo-var. $Gal(MT(\mathbb{Z}), dR)$

MZVs

upper bound $\dim_{\mathbb{Q}} Z_n \leq d_n$ t_n

$d_{n-1} + d_n$

comes from π_1^M

$Gal(MT(\mathbb{Z}), dR) \xrightarrow{?} Aut\, \pi_1^{dR}(V...)$

$\rightsquigarrow$ (Brown, motivic Belyi)

$d_{n+3} = d_{n+1} + d_n$

meaning of $\{d_n\}_{n\geq 0}$

generating fct $\sum_{n\geq 0} d_n t^n = \frac{1}{1-t^2-t^3}$

$= \frac{1}{1-t^2} \frac{1}{1-\frac{t^3}{1-t^2}}$

$= \frac{1}{1-t^2} \frac{1}{1-(t^3+t^5+t^7+\cdots)}$

Gal(MT(Q), dR)
$\Vert$
$G_m \ltimes U_{dR}$
π^2 mt:2

$\mathrm{rk}\, K_{2n-1}(\mathbb{Q}) = \begin{cases} 0 & n: \text{even or } =1 \\ 1 & n: \text{odd}, \neq 1 \end{cases}$

coeff of $t^1 = 0$
$t^2 = 0$
$t^3 = 1$
$t^4 = 0$
$t^5 = 1$
...

$U(\mathrm{Lie}\, U_{dR})$
# of generators
-series

Rem) $\exists$ p-adic analogue of
the above conj. (& Rem)

Gal(MT(Q), dR)

K-theory
$K_{2n-1}(\mathbb{Z})_{\mathbb{Q}}$

$\mathrm{comp}_{dR,B}$

Rem multiple L-values

$MT(\mathbb{Z}[\mu_N, \frac{1}{1-\mu}|_{\mu\in\mu_N}])$

$\rightsquigarrow$ rk $K_{2n-1}(\mathbb{Z}[\mu_N, \frac{1}{1-\lambda}|_{\lambda\in\mu_N}])$ size

rk $K_{2n-1}(\mathbb{Z}) = \begin{cases} 0 & n: \text{even or } =1 \\ 1 & n: \text{odd}, \neq 1 \end{cases}$

$\mathcal{U}(\mathrm{Lie}\, \mathcal{U}_{dR})$

# of generators

series

coeff of $t^1 = 0$

$t^2 = 0$

$t^3 = 1$

$t^4 = 0$

$t^5 = 1$ ...

Conj (Grothendieck)

$Gal(MT(\mathbb{Z}), dR)(\mathbb{C}) \ni comp_{dR,B}$ is $\mathbb{Q}$-Zariski dense

(i.e. $comp^*_{dR,B} : \Gamma(Gal(MT(\mathbb{Z}), dR), \mathcal{O}) \longrightarrow \mathbb{C}$ inj)

Lem the above conj. implies $\begin{cases} \text{Zagier's dimension conj.} \\ \text{irrational conj.} \end{cases}$

$Gal(MT(\mathbb{Z}), dR) \xrightarrow{?} Aut\, \pi_1^{dR}(\mathbb{P}^1 \setminus \{0,1,\infty\}, \vec{01})$ too big

K-theory $K_{2n-1}(\mathbb{Z})_{\mathbb{Q}}$

$comp_{dR,B} \longmapsto \Phi_{KZ}$ Drinfel'd assoc.

upper bound

$\dim Z_n \leq d_n$

Rem) $\exists$ p-adic analogue of the above conj. (& Rem)

(Y.)

(the p-adic dim. conj.:

$d_{n+3} = d_{n+1} + d_n$

$d_0 = 1,\ d_1 = 0,\ d_2 = 0$)

⇕

" $2\pi i = 0$ in the p-adic world"

from d t_{dR}